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**CE-580**

**COMPUTATIONAL TECHNIQUES**

**FOR**

**FLUID DYNAMICS**

**HOMEWORK #1**

**Transient Flow in Pipe-Reservoir System**

**Taha Yaşar Demir**

**1881978**

# CALCULATION

Assumptions:

* Incompressible Flow
* Height between point 1 and 2 is constant
* Flow is fully developed everywhere between point 2 and 3

Dimensions and flow properties:

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|  |
| *Figure 1: Control Volume* |

Momentum equation

Writing mass balance over the domain

Where

we assumed constant velocity along the pipe so momentum fluxes will cancel each other, and we can take velocity constant term out and take the integration

Simplified force balance equation

Assuming quasi-steady flow we can use Darcy’s friction factor

Momentum equation becomes

For pressure at point 2 we can use Bernoulli equation

After arranging we get the time rate of change of velocity

The time derivative can be expressed using first order forward differencing

Final momentum equation

Where f is friction factor and may be obtained from the Swamee-Jain formula

And is the Reynolds number

The steady-state velocities cannot be calculated due to the Reynolds number term in the friction factor, so they are calculated numerically

Convergence criteria is set to and defined as

# RESULTS

|  |  |  |  |
| --- | --- | --- | --- |
| **L (m)** | **T\_s (s)** | **V(T\_s) (m/s)** | **V\_s (m/s)** |
| **50** | **16** | **6.57997465** | **6.58654213** |
| **100** | **24.4** | **4.9966011** | **5.00158691** |
| **200** | **35.98** | **3.66563702** | **3.66929102** |
| **500** | **53.78** | **2.35879743** | **2.3605597** |
| *Table 1: Time required to reach steady state condition with respect to pipe lengths* | | | |

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|  |
| *Figure 2: Change of average velocity with respect to time* |

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|  |
| *Figure 3: Effect of time step when L=50 m* |

# DISCUSSION

Steady state velocity implies that the time derivative will be zero,

But the friction factor contains Reynolds number and it is calculated using steady state velocity, too. Even though every other term is known, steady state velocities cannot be calculated analytically. On the other hand, it is easy to calculate numerically. The steady state velocity results in the *Table 1* have at least 8 digits accuracy and they all converged in our time domain except the case where L=500 m. When L=500 m steady state velocity converges at time about 90 second with time step 0.02.

The reason behind the steady state velocity variation in the *Figure 3* is that the shear force is directly proportional to the surface area. As the pipe length increases the surface area increases and the net force acting on the fluid becomes smaller. This results in slower velocities and slower velocity means less mass flow rate (assuming incompressible fluid).

In addition, with the increasing pipe length, the amount of fluid in the pipe increases. In other words, the more pipe length, the more mass in the pipe. lower net force tries to accelerate more mass, resulting in slower velocities. Furthermore, this is also the reason of longer convergence times of different pipe lengths.

To decrease computational effort, we may use larger time steps. In this case, first order scheme overpredict the results. Taking larger steps will result in larger overprediction and eventually solution will diverge as seen in *Figure 3*. Which means the convergence criteria may never be met. Different high order accuracy schemes with large time steps can be used to prevent the oscillations in the *Figure 3.*

# Appendix

Data are plotted using GNU-Plot

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| parameter (mx=5000)  real H,D,k\_s,nu,rho,L,g  real f, Re  real inc  data eps/1.0E-3/ L/50./  real Tf,dt  real V(mx),T(mx)  character string\*5,filename\*9,len\*3  integer index  c..Arrange the output file name  write(string,'(f5.3)') L/1000  read(string,'(2x,a3)') len  filename = 'L-'//len//'.dat'  open(1,file=filename)  c..Final time and time step  Tf = 60  dt = 0.02  index = Tf/dt+1  c..Flow parameters and dimensions  H = 8  D = 0.3  k\_s = 1.0E-4  nu = 1.0E-3  rho = 1000  g = 9.81  c..Initial Velocity and Time  V(1) = 0.  T(1) = 0.  c..Loop to calculate steady-state velocities  do i=1,index  Re = (rho\*V(i)\*D)/nu  f = 0.25/((log10((k\_s/(3.7\*D))+(5.74/(Re\*\*0.9))))\*\*2)  inc = H-((1+f\*(L/D))\*((V(i)\*\*2)/(2\*g)))  V(i+1) = V(i) + dt\*inc\*(g/L)  T(i+1) = T(i) + dt  write(1,'(2f15.10)') V(i),T(i) ! get output data  enddo  c..Output the steadt-state vel. and corresponding time  print\*, "Steady vel"," time"  print\*, V(index),T(index)  close(1)  c..Find the convergence time and speed  k=0  eps1=1  do while(eps1.gt.eps)  k=k+1  eps1 = (V(index)-V(k))/V(index)  enddo  print\*, k, T(k), V(k)  stop  end |
| Fortran Code used in Calculations |